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Equation of State of Protoneutron Star Matter

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We have calculated the equation of state of protoneutron star matter by using the lowest order constrained variational method. In our calculations, the modern Argonne potential (AV_{18}) together with its older model potential (AV_{14}) are used. It is found that the equation of state for high lepton fraction is stiffer than for low lepton fraction. It is seen that the increasing effect of pressure due to high lepton fraction and due to entropy are comparable. It is shown that the temperature and adiabatic index depend on the values of both entropy and lepton fraction.

1. INTRODUCTION

Neutron stars are among the most fascinating objects in our universe. They contain over a solar mass of matter within a radius of ∼10 km. Therefore, neutron stars have high densities of order 10^{15} g/cm^3 (Heiselberg and Pandharipande, 2000; Lattimer and Prakash, 2000a; Shapiro and Teukolsky, 1983).

Just after gravitational collapse of a massive stellar core, a protoneutron star is formed. Protoneutron stars have almost constant entropy per nucleon of order $1 - 2k_B$ (k_B is the Boltzmann constant) (Bethe *et al.*, 1979; Burrows and Lattimer, 1986; Epstein and Pethick, 1981; Pons *et al.*, 1999). Furthermore, protoneutron star matter is characterized by high and nearly constant lepton fraction of order 0.3–0.4. This is due to the fact that neutrinos are trapped in the protoneutron star matter (Bethe *et al.*, 1979; Burrows and Lattimer, 1986; Epstein and Pethick, 1981; Pons *et al.*, 1999). We can realize that the protoneutron star matter are quite different from ordinary neutron star matter which is cold and also has very low lepton fraction. In this article, we neglect neutrinos and consider an uncharged matter composed of neutrons, protons, and electrons, since neutrino fraction is expected to be small (Bethe *et al.*, 1979; Lattimer *et al.*, 1985). In protoneutron star matter, the muons number density is small (Fukuda *et al.*, 1998) and therefore, they are neglected here.

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The equation of state of protoneutron star matter over a wide range of densities is the basic input quantity for solving the structure equations (Lattimer *et al.*, 1991; Prakash, 1994; Prakash *et al.*, 1988). The equation of state of neutron star matter chiefly depends on the nucleon–nucleon interaction. In recent years, various potential models have been used for calculating the properties of neutron star matter. The results of these calculations show large differences. It is shown that by using the modern potentials, these differences become small (Engvik *et al.*, 1997). In present work, we use the new Argonne potential (AV_{18}) (Wiringa *et al.*, 1995) which fits the pp and np scattering data with high precision.

In this paper, we intend to calculate the equation of state of protoneutron star matter and some of its properties with the AV_{18} and AV_{14} (Wiringa *et al.*, 1984) potentials by using the lowest order constrained vaiational (LOCV) method (Bordbar, 2000, 2001; Bordbar and Modarres, 1997, 1998; Bordbar and Riazi, 2001, in press; Modarres and Bordbar, 1998).

2. FORMALISM OF CALCULATIONS

For protoneutron star matter, we write the total energy per nucleon as the sum of contributions from leptons and nucleons:

$$E = E_{\rm lep} + E_{\rm nucl}. \tag{1}$$

The charge neutrality condition imposes the following constraint in our calculations:

$$Y_{\rm l} = Y_{\rm p}, \tag{2}$$

where $Y_{\rm l}$ and $Y_{\rm p}$ are the lepton and proton fractions, respectively.

Now, we discuss the calculation of energy of leptons ($E_{\rm lep}$) and nucleons ($E_{\rm nucl}$) separately.

2.1. Leptons

We can ignore the electromagnetic interaction, since the protoneutron star matter is electrically neutral. Therefore, the contribution from the energy of leptons is

$$E_{\rm lep} = \frac{m^4 c^5}{\pi^2 \rho \hbar^3} \int_0^\infty n(x)\sqrt{1+x^2}\,x^2\,dx, \tag{3}$$

where ρ is the total number density, which is given by Eq. (9), and x is

$$x = \frac{\hbar k}{mc}. \tag{4}$$

In Eq. (3), the Fermi–Dirac distribution function $n(x)$ is defined as

$$n(x) = [1 + \exp\{\beta\,[\epsilon(x) - \mu]\}]^{-1}, \tag{5}$$

where $\beta = \frac{1}{k_B \mathcal{T}}$ ($\mathcal{T}$ is temperature) and

$$\epsilon(x) = mc^2\sqrt{1 + x^2}. \tag{6}$$

2.2. Nucleons

We calculate the contribution from the energy of nucleons by using LOCV method (Bordbar, 2000, 2001; Bordbar and Modarres, 1997, 1998; Bordbar and Riazi, 2001, in press; Modarres and Bordbar, 1998). We consider up to the two-body term in the cluster expansion for the energy functional

$$E_{\text{nucl}} = E_1 + E_2. \tag{7}$$

The one-body energy E_1 is

$$E_1 = \sum_{i=n,p} \frac{\hbar^2}{2m_i \rho \pi^2} \int_0^\infty n_i(k) k^4\, dk, \tag{8}$$

where ρ is given by

$$\rho = \sum_{i=n,p} \frac{1}{\pi^2} \int_0^\infty n_i(k) k^2\, dk, \tag{9}$$

and $n(k)$ is defined as

$$n(k) = [1 + \exp\{\beta\,[\epsilon(k) - \mu]\}]^{-1}, \tag{10}$$

where

$$\epsilon(k) = \frac{\hbar^2 k^2}{2m}. \tag{11}$$

The two-body energy E_2 is

$$E_2 = (2A)^{-1} \sum_{ij} \langle ij| \left\{ -\frac{\hbar^2}{2m} [\,f(12), [\nabla_{12},\ f(12)]] + f(12) V(12) f(12) \right\} |ij\rangle_a, \tag{12}$$

where $f(12)$ and $V(12)$ are the two-body correlation function and nucleon–nucleon potential, respectively. $V(12)$ has the following general form (Wiringa *et al.*, 1995):

$$V(12) = \sum_{p=1}^{18} V^p(r_{12}) O_{12}^p. \tag{13}$$

The same subscript on two-body matrix element means that the independent particle bra and ket involved are antisymmetric.

As in our previous works, we minimize the two-body energy E_2 with respect to the variation in the two-body correlation function but subject to the normalization constraint (Bordbar, 2000, 2001; Bordbar and Modarres, 1997, 1998; Bordbar and Riazi, 2001, in press; Modarres and Bordbar, 1998). The normalization constraint introduces a Lagrange multiplier. From this minimization, we get a set of

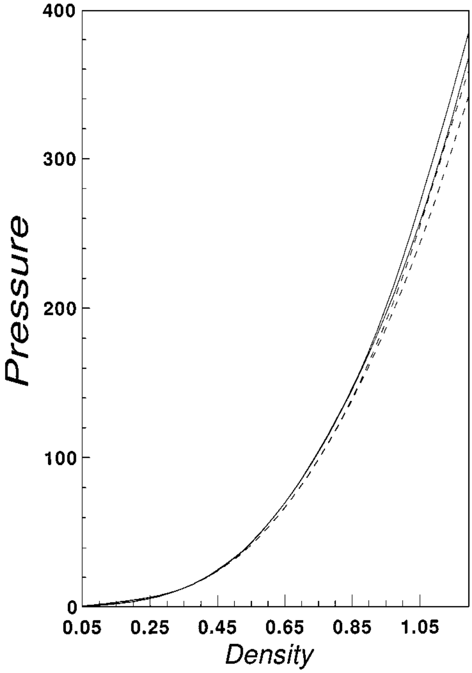

Fig. 1. The protoneutron star matter equation of state with the AV_{18} (full curves) and AV_{14} (dashed curves) potentials at $s = 1.0$ (lower curves) and 2.0 (upper curves) for $Y_l = 0.3$.

Euler–Lagrange differential equations, which are the same as given in our previous papers (Bordbar and Modarres, 1998). The correlation functions are calculated by solving the differential equations and then the two-body energy E_2 is computed.

3. RESULTS AND DISCUSSION

In Figs. 1 and 2, we have shown the equation of state of protoneutron star matter with the AV_{18} and AV_{14} potentials at different values of entropy ($s = 1.0, 2.0$) for lepton fractions $Y_l = 0.3, 0.4$. It is seen that the increasing of pressure because of the increasing of entropy for $Y_l = 0.3$ is more sensitive than for $Y_l = 0.4$. It is also seen that for $Y_l = 0.4$, the equations of state with the AV_{18} and AV_{14} potentials become nearly identical. This is because with increasing lepton fraction (Y_l), the energy contribution from leptons becomes more important and it dominates at high Y_l. In order to clarify this behavior, our results for equation of state in two different physical cases of the neutron star matter are compared in

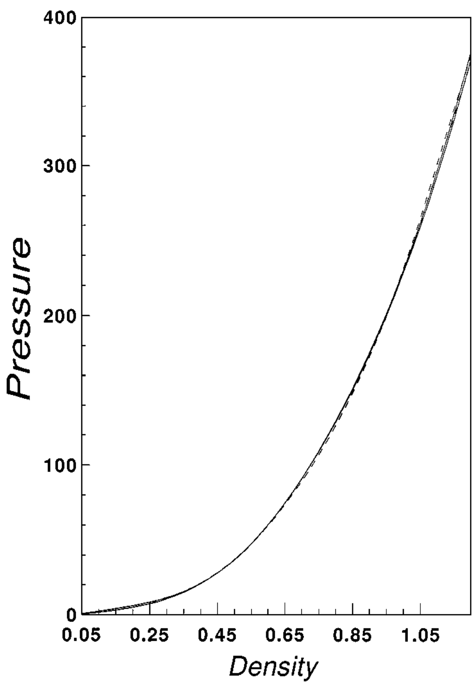

Fig. 2. As Fig. 1 but for $Y_l = 0.4$.

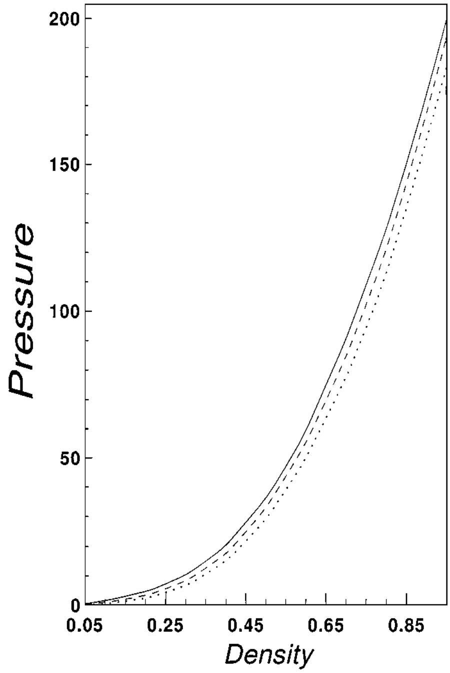

Fig. 3. Comparison of our results for the equation of state (with the AV_{18} potential) at $s = 1.0$ for $Y_l = 0.4$ (full curves) and 0.3 (dashed curves) with the case of untrapped neutrinos at $s = 1.0$ (Bordbar and Riazi, in press) (dotted curves).

Fig. 3. The first one is the matter with untrapped neutrinos (low lepton fraction) at $s = 1.0$ (Bordbar and Riazi, in press) and the other one is the matter at $s = 1.0$ with $Y_l = 0.3, 0.4$. We see that the pressure increases by increasing lepton fraction (Y_l), especially at high densities, and the equation of state for high Y_l is stiffer than for low Y_l. The lepton energy plays a dominant role in the stiffening of the equation of state of protoneutron star matter. This means that the stiffening effect due to the high lepton fraction overwhelms the softening effect due to the high proton fraction. In our previous works, we have shown that the pressure of nucleonic matter decreases by increasing proton fraction (Bordbar, 2000, 2001). This result is of crucial importance for determining the mass of neutron star (Lattimer and Prakash, 2000b).

In Fig. 4, we have presented the temperature of protoneutron star matter versus density with the AV_{18} potential at $s = 1.0, 2.0$ for $Y_l = 0.3, 0.4$. In this figure, we have also shown our previous results for temperature in the case of untrapped

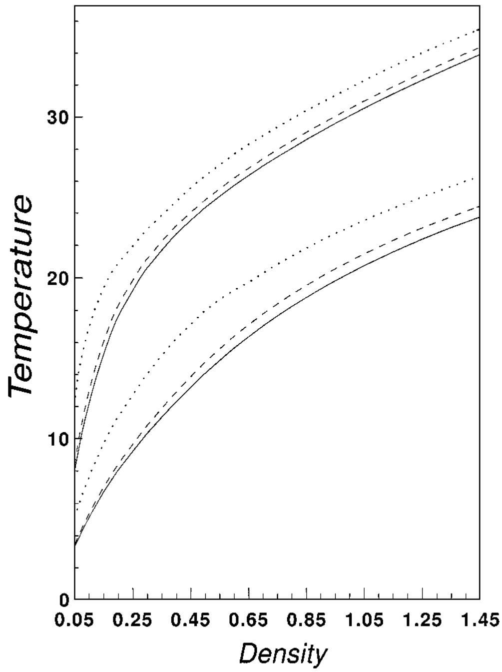

Fig. 4. Temperature of protoneutron star matter as a function of density with the AV_{18} for $Y_l = 0.3$ (dashed curves) and 0.4 (full curves) at $s = 1.0$ (lower curves) and 2.0 (upper curves). Our results in the case of untrapped neutrinos (Bordbar and Riazi, in press) (dotted curves) are given for comparison.

neutrinos (low lepton fraction) (Bordbar and Riazi, in press) for comparison. Our results for temperature with the AV_{14} potential are very similar to those with the AV_{18} potential. We see that the temperature increases by increasing entropy. It is seen that the temperature decreases by increasing lepton fraction. This is because at high lepton fraction, the proton fraction and neutron fraction become more equal. In our previous paper, it is shown that the temperature of nucleonic matter decreases by increasing proton-to-neutron ratio (Bordbar, 2000).

We know that the stability of a star depends on the value of adiabatic index Γ (Shapiro and Teukolsky, 1983). The adiabatic index Γ can be calculated from equation of state of protoneutron star matter by using the following equation:

$$\Gamma = \frac{\rho}{P}\left(\frac{\partial P}{\partial \rho}\right)_{Y_l,s}. \tag{14}$$

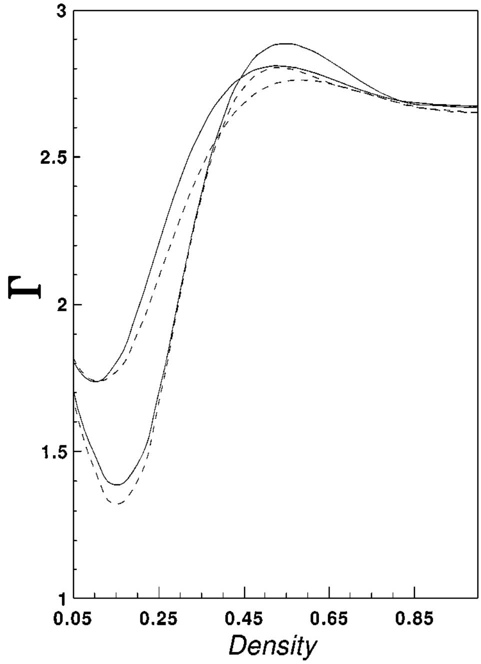

Fig. 5. Adiabatic index as a function of density with the AV_{18} (full curves) and AV_{14} (dashed curves) potentials at $s = 1.0$ (upper curves) and 2.0 (lower curves) for $Y_l = 0.3$.

In Figs. 5 and 6, we have plotted our results for Γ with the AV_{18} and AV_{14} potentials at $s = 1.0$, 2.0 for $Y_l = 0.3, 0.4$. It can be seen that Γ decreases by increasing both entropy and lepton fraction. In order to clarify the explicit dependence of Γ on lepton fraction, a comparison between our results (with the AV_{18} potential) in the case of untrapped neutrinos (low lepton fraction) at $s = 1.0$ (Bordbar and Riazi, in press) and in the case of $Y_l = 0.3, 0.4$ at $s = 1.0$ is made in Fig. 7.

The sound velocity v_s in the units of c (speed of light),

$$v_s = \sqrt{\Gamma \frac{P}{P + e}}, \tag{15}$$

where

$$e = \rho(E + mc^2), \tag{16}$$

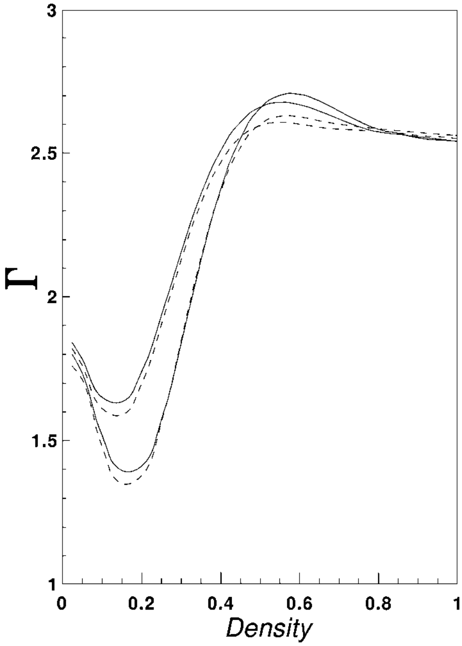

Fig. 6. As Fig. 5 but for $Y_l = 0.4$.

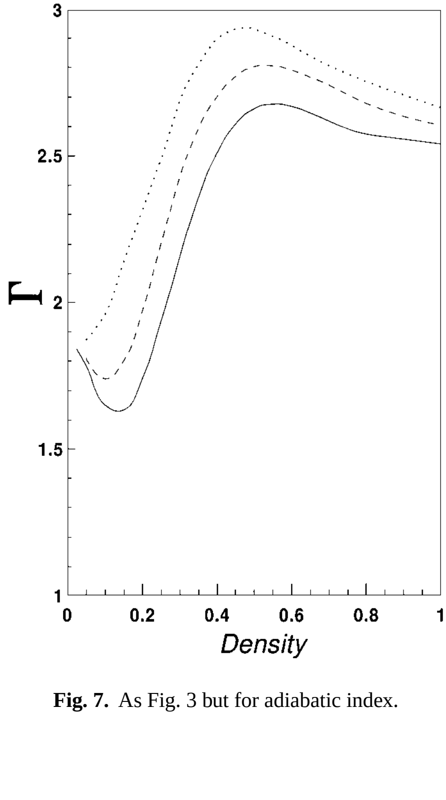

Fig. 7. As Fig. 3 but for adiabatic index.

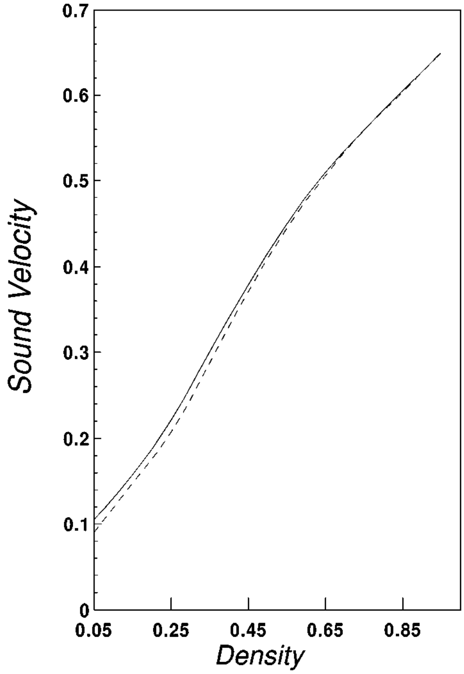

Fig. 8. Sound velocity (in the units of c) versus density with the AV_{18} potential for $Y_l = 0.4$ at $s = 1.0$ (dashed curves) and 2.0 (full curves). Our results for $Y_l = 0.3$ and also with the AV_{14} potential are nearly identical to those shown in this figure.

is given in Fig. 8. We see that the sound velocity increases by increasing density, mean-while v_s is always smaller than c. Therefore, our calculated equations of state of protoneutron star matter are causal.

4. SUMMARY AND CONCLUSION

For protoneutron star matter, we have considered an electrically neutral composition of neutrons, protons, and electrons. We have used the LOCV method for calculating the equation of state of protoneutron star matter and some of its properties over a wide range of densities at different values of entropy for different lepton fractions with the AV_{18} and AV_{14} potentials. It was seen that at high lepton fraction, the energy contribution from leptons dominates and the equation of state of protoneutron star matter is stiffer than at low lepton fraction. We have found that the temperature increases by increasing entropy and decreases by increasing lepton fraction. We have also found that the adiabatic index decreases by increasing both entropy and lepton fraction. It was shown that our results for equation of state of protoneutron star matter obey the causality condition.

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